Parametric four-wave mixing in atomic vapor induced by a frequency-comb and a cw laser

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The objective is to study the coherent blue light generated in rubidium vapor by four wave mixing due to the combined action of an ultrashort pulse train and a cw diode laser.
Outline

1. Lasers
2. Atomic system
3. Four-wave mixing
4. Correlated work
5. Setup
6. Experimental Results
7. Theory
8. Conclusions
Diode laser
Diode laser

- Power = 30 mW
- Linewidth < 1 MHz
- $\lambda = 780 \text{ nm}$
- Scan frequency = 10 GHz
Diode laser

- Power = 30 mW
- Linewidth < 1 MHz

\[ E(t) = E_0 e^{i\omega_{cw} t} \]

- \( \lambda = 780 \) nm
- Scan frequency = 10 GHz
Femtosecond laser

\[ E(t) = E_0 N - 1 \sum_{n=0}^\infty \text{sech} \left[ \frac{1.76(t-nT_R)}{T_p} \right] e^{\text{in} \left( \omega_{fs} T_R - \phi \right) + i\omega_{fs} t} \]
Femtosecond laser

- Average power = 500 mW
- Peak power = 50 kW
- Repetition rate = 76 MHz
- Temporal width = 150 fs
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Femtosecond laser

\[ E(t) \]

\[ \Delta \phi \]

\[ 2\Delta \phi \]

\[ (a) \]

\[ T_R \]

\[ \omega_c \]

\[ 1 / T_R \]

Transformada de Fourier

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Atomic system

85 Rb

- 5D<sub>5/2</sub>
- 5D<sub>3/2</sub>
- 5P<sub>3/2</sub>
- 5P<sub>1/2</sub>
- 5S<sub>1/2</sub>

F' = 1
F' = 2
F' = 3
F' = 4
F' = 5

F" = 0
F" = 1
F" = 2
F" = 3
F" = 4
F" = 5

0.2 nm
776 nm
780 nm
795 nm

88 888 MHz
386 252 117 MHz
7 122 757 MHz
377 105 910 MHz

Frequência (MHz)
Transmissão (V)
Tensão (V)

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### Atomic system

#### $^{85}$Rb

- **$5D_{3/2}$**
  - 3 MHz
  - 5 MHz
  - 8 MHz
  - 9 MHz
  - 9 MHz
- **$5D_{5/2}$**
  - 19 MHz
  - 12 MHz
  - 7 MHz
- **$5P_{3/2}$**
  - 121 MHz
  - 63 MHz
  - 29 MHz
- **$5P_{1/2}$**
  - 362 MHz
  - 7 MHz
- **$5S_{1/2}$**
  - 3036 MHz

#### Frequencies

<table>
<thead>
<tr>
<th>Transition</th>
<th>Frequency (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F'' = 0$</td>
<td>3 MHz</td>
</tr>
<tr>
<td>$F'' = 1$</td>
<td>5 MHz</td>
</tr>
<tr>
<td>$F'' = 2$</td>
<td>8 MHz</td>
</tr>
<tr>
<td>$F'' = 3$</td>
<td>9 MHz</td>
</tr>
<tr>
<td>$F'' = 4$</td>
<td>9 MHz</td>
</tr>
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<tr>
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</tr>
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</tbody>
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#### Transmissions

- **$(a)$**
  - $^{85}$Rb, $F = 1$
  - $^{85}$Rb, $F = 2$

- **$(b)$**
  - $^{87}$Rb, $F = 3$
  - $^{87}$Rb, $F = 2$

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Four-wave mixing is a nonlinear process that involves four electromagnetic waves.
Four-wave mixing

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- In general, we have three incident beams and one generated beam.
Four-wave mixing

- Four-wave mixing is a nonlinear process that involves four electromagnetic waves.
- In general, we have three incident beams and one generated beam.
- In the weak interaction limit, it is a third-order process, where the generated field can be expressed as

\[ P_4 = \epsilon_0 \chi^{(3)} E_1 E_2 E_3 \]
Driven nonlinear oscillator: classical analysis

Nonlinear spring:
\[ F_{el} = -kx - k_2 x^2 \]

\[ F_{ext}(t) = F_0 (\cos \omega_1 t + \cos \omega_2 t) \]
Driven nonlinear oscillator: classical analysis

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Wave-mixing

\[
\omega_1, \omega_2 \rightarrow \begin{cases} 
\omega_1 \\
\omega_2 \\
2\omega_1 \\
2\omega_2 \\
\omega_1 + \omega_2 \\
\omega_1 - \omega_2
\end{cases}
\]
Wave-mixing

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\end{cases}
\]

General formula:

\[
\omega_1, \omega_2, \omega_3, \ldots \rightarrow \alpha_1 \omega_1 \pm \alpha_2 \omega_2 \pm \alpha_3 \omega_3 \pm \cdots
\]
Wave-mixing

\[ \omega_1, \omega_2 \rightarrow \{ \omega_1, \omega_2, 2\omega_1, 2\omega_2, \omega_1 + \omega_2, \omega_1 - \omega_2 \} \]

General formula:

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Electromagnetism:

\[ P = \varepsilon_0 \chi^{(1)} E + \varepsilon_0 \chi^{(2)} E^2 + \varepsilon_0 \chi^{(3)} E^3 + \cdots \]

\[ \chi^{(1)} \approx 1 \]
\[ \chi^{(2)} \approx 10^{-12} \text{ m/V} \]
\[ \chi^{(3)} \approx 10^{-24} \text{ m}^2/V^2 \]
Coherent and collimated blue light generated by four-wave mixing in Rb vapour

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Correlated work

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\[ k_{\text{IR}} + k_1 + k_2 = k_{\text{IR}} + k_{\text{BL}} \]

\[ \text{776 nm, 780 nm, 420 nm} \]
Setup

Ti:sapphire laser
500 mW
76 MHz

Diode laser
20 mW
CW

Counter

Sat. Abs.

PBS

M

L

M

M

Rb

L

F

5.2 µm
(generated)

5D_{5/2}

776 nm
Ti:sapph

6P

5P_{3/2}

420 nm
(generated)

780 nm
Diode

5S_{1/2}

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12 / 25
Mesa óptica
Experimental Results

- $I_{cw} = 1.9 \ \text{W/cm}^2$
- $I_{fs} = 1.0 \ \text{mW/cm}^2$ (each mode)
- $T = 85^\circ\text{C}$
Experimental Results

- $I_{cw} = 1.9 \text{ W/cm}^2$
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- Broad peaks: fluorescence induced by both lasers
Experimental Results

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- $I_{fs} = 1.0$ mW/cm$^2$ (each mode)
- $T = 85^\circ$C
- Broad peaks: fluorescence induced by both lasers
- Flat Background: fluorescence induced by femtosecond laser

![Graph showing FWM in Rb vapor](image-url)
Polarization and density dependence

![Graph showing polarization and density dependence](image)
Experimental Results

Polarization and density dependence

![Graph showing polarization and density dependence](image)

Blue intensity (arb. units) vs. Diode frequency (GHz)

- Blue fluorescence at 90°
- Parallel polarization
- Perpendicular polarization

Rubidium vapor

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Experimental Results

Polarization and density dependence

Blue intensity (arb. units)

Diode frequency (GHz)

Blue fluorescence at 90°

parallel polarization

perpendicular polarization

Rubidium vapor

(a) $^{87}\text{Rb}, F_g = 2$

(b) $^{85}\text{Rb}, F_g = 3$

(c) $82 °C$

(d) $87 °C$

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FWM in Rb vapor

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Experimental Results

- $T = 85^\circ C$
- Slow scanning
- Average of 10 measurements

![Graph showing experimental results for $^{85}\text{Rb, } F_g = 3$]
Experimental Results

- $T = 85^\circ C$
- Slow scanning
- Average of 10 measurements
- Peak linewidths: $\approx 55$ MHz

![Graph showing FWM in Rb vapor with $^{85}\text{Rb, } F_g = 3$)](image-url)
Experimental Results

- $T = 85^\circ\text{C}$
- Slow scanning
- Average of 10 measurements
- Peak linewidths: $\approx 55$ MHz
- Frequency difference between two adjacent peaks: $78 \pm 4$ MHz
Density dependence

- Femtosecond and diode lasers in resonance with your transitions

![Graph showing exponential growth and threshold shift as a function of diode intensity.

Atomic density (10^{12} cm^{-3}) vs. PFWM signal amplitude (arb. units).

I_{cw} = 1.9 \text{ W/cm}^2

I_{cw} = 9.4 \text{ W/cm}^2

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Density dependence

- Femtosecond and diode lasers in resonance with your transitions
- Exponential growth
Density dependence

- Femtosecond and diode lasers in resonance with your transitions
- Exponential growth
- “Threshold” shift as a functions of the diode intensity
Diode intensity dependence

![Graph showing diode intensity dependence](image)

- Diode power:
  - 0.5 mW
  - 2 mW
  - 4 mW
  - 6 mW
  - 8 mW
  - 10 mW

- Blue intensity (arb. units) vs. Diode frequency (MHz)

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Diamond-type four-level system interacting with four cw lasers.

- $\Omega_1$: diode laser
- $\Omega_2$: one mode of the femtosecond laser
- $\Omega_3$: seed field
- $\Omega_4$: blue beam generated
Diamond-type four-level system interacting with four cw lasers.

- $\Omega_1$: diode laser
- $\Omega_2$: one mode of the femtosecond laser
- $\Omega_3$: seed field
- $\Omega_4$: blue beam generated

\[ \hat{H}_{int} = -\hat{\mu} \cdot E \]

(Dipole electric approximation)
Maxwell-Bloch equations

\[ \frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar} \left[ \hat{H}, \hat{\rho} \right] \]  
(Liouville-Neumann)

\[ \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P}{\partial t^2} \]  
(Wave equation)

\[ P = N \langle \hat{\mu} \rangle \]  
(Polarization)
Maxwell-Bloch equations

\[
\frac{\partial \rho_{jk}(z, t)}{\partial t} = -(i\omega_{jk} + \gamma_{jk})\rho_{jk}(z, t) - \frac{i}{\hbar} \langle j | [\hat{H}_{\text{int}}, \hat{\rho}] | k \rangle \\
\frac{\partial \Omega_j(z, t)}{\partial z} = -i\alpha_{jk}\gamma_{jk}\sigma_{jk}(z, t)
\]

where

\[
\Omega_j = \frac{\mu_{jk} E_0^j}{\hbar} \quad \text{(Rabi frequency)}
\]

\[
\alpha_{jk} = \frac{\mu_{jk}^2 \omega_j}{2\hbar c \epsilon_0 \gamma_{jk}} N
\]

\[
\sigma_{jk} = \rho_{jk} e^{-i\omega_j t}
\]
Theoretical result

- Numerical solution of the equations: Runge-Kutta method
- All field at resonance
- Initial conditions: $\Omega_2 = 2.4\gamma_{33}$, $\Omega_3 = 2.4 \times 10^{-7}\gamma_{33}$ and $\Omega_4 = 0$. 

![Graph showing $\Omega_1$ (normalized) vs $\alpha_{12}Z$ and $\Omega_4$ (rad/s) vs $\alpha_{12}Z$ with lines for Diode and Blue.](image)
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\[\Omega_1 \text{ (normalized)} \]

\[\Omega_4 \text{ (rad/s)} \]

\[\alpha_{12}Z \]

\[\rho_{11}, \rho_{22}, \rho_{33}, \rho_{44} \]
Theoretical result

\[ \Omega_1 = 0.7 \gamma_{22} \]

\[ \Omega_1 = 1.4 \gamma_{22} \]
We have investigated the coherent blue light generated in atomic vapor using a parametric four-wave mixing process due to the combined action of a cw laser and a train of ultrashort pulses.

Each individual mode is responsible for inducing the nonlinear process.

Blue signal characterization: exponential growth with saturation.

The density dependence was theoretically modeled.

Next step: frequency dependence modeling (with CUDA!).
Thank you very much!