Geometria Analítica - Prova 1

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27 de dezembro de 2021

Início: 19:00 - duração: 2:30 horas

Só serão consideradas as respostas que forem devidamente justificadas.

Questão 01: Operações com vetores

Dados os vetores $\vec{u} = 2\hat{\imath} - 5\hat{\jmath} + \hat{k}$, $\vec{v} = -3\hat{\imath} + 2\hat{\jmath}$ e $\vec{w} = \frac{1}{2}\hat{\jmath}$, e sabendo que $\alpha = 10$, efetue as operações abaixo:

(a) **(0,7)** $2\vec{u} - 3\vec{v}$

(b) **(0,7)** $\alpha(\vec{u} \cdot \vec{v})$

(c) **(0,7)** $\vec{v} \times \vec{w}$

(d) **(0,7)** $|\vec{u}|$

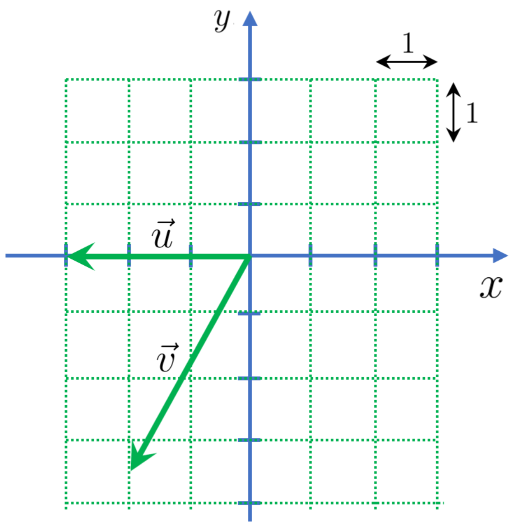
(e) **(0,7)** $(\vec{u}, \vec{v}, \vec{w})$

Questão 02: (2,5)

Calcule a distância do ponto $P(2, 3, 6)$ à reta que passa pelos pontos $A(1, 0, 0)$ e $B(-2, 1, 3)$.

Questão 03: (2,0)

Na figura abaixo, o ângulo formado pelos vetores $\vec{u}$ e $\vec{v}$ vale 60º. Calcule a componente y do vetor $\vec{v}$.



Questão 04: (2,0)

Calcule a área do paralelogramo determinado pelos vetores $\vec{a} = (1, 0, -1)$ e $\vec{b} = (5, 1, 3)$.

GABARITO

1 – $\vec{u} = 2\hat{i} - 5\hat{j} + \hat{k}$

$\vec{v} = -3\hat{i} + 2\hat{j}$

$\vec{w} = \frac{1}{2}\hat{j}$

$\alpha = 10$

A) $2\vec{u} - 3\vec{v} =$

$= 2(2, -5, 1) - 3(-3, 2, 0)$

$= (4, -10, 2) + (9, -6, 0)$

$= (13, -16, 2)$

$$\boxed{2\vec{u} - 3\vec{v} = 13\hat{i} - 16\hat{j} + 2\hat{k}}$$

B) $\alpha(\vec{u} \cdot \vec{v}) = 10(2, -5, 1) \cdot (-3, 2, 0)$

$= 10(-6 - 10 + 0)$

$$\boxed{\alpha(\vec{u} \cdot \vec{v}) = -160}$$

C) $$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 0 \\ 0 & \frac{1}{2} & 0 \end{vmatrix}$$

$= \hat{i} \cdot 0 + \hat{j} \cdot 0 + \hat{k} \cdot \left(-\frac{3}{2}\right)$

$$\boxed{\vec{v} \times \vec{w} = -\frac{3}{2}\hat{k}}$$

D) $|\vec{u}| = \sqrt{2^2 + (-5)^2 + 1^2}$

$= \sqrt{30}$

$$\boxed{|\vec{u}| = \sqrt{30}}$$

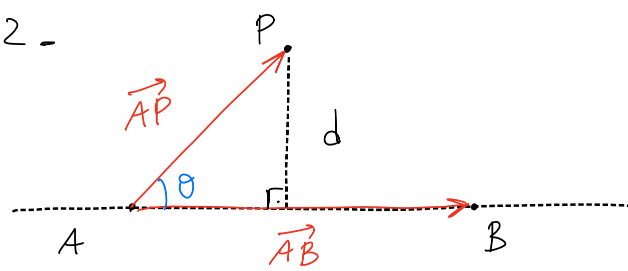
E)

$$(\vec{u}, \vec{v}, \vec{w}) = \begin{vmatrix} 2 & -5 & 1 \\ -3 & 2 & 0 \\ 0 & 1/2 & 0 \end{vmatrix} \begin{matrix} 2 & -5 \\ -3 & 2 \\ 0 & 1/2 \end{matrix}$$

$$= 0 + 0 - \frac{3}{2} - 0 - 0 - 0$$

$$\boxed{(\vec{u}, \vec{v}, \vec{w}) = -\frac{3}{2}}$$

2.-



$$\vec{AB} = (-3, 1, 3)$$
$$\vec{AP} = (1, 3, 6)$$
$$|\vec{AB}| = \sqrt{19}$$
$$|\vec{AP}| = \sqrt{1+9+36} = \sqrt{46}$$

MÉTODO 1:

$$\cos\theta = \frac{\vec{AB} \cdot \vec{AP}}{|\vec{AB}|\,|\vec{AP}|}$$

$$\cos\theta = \frac{-3+3+18}{\sqrt{19}\,\sqrt{46}}$$

$$\cos\theta = \frac{18}{\sqrt{19}\,\sqrt{46}}$$

$$\operatorname{sen}\theta = \frac{d}{|\vec{AP}|}$$

$$d = \sqrt{1-\cos^2\theta}\,|\vec{AP}|$$

$$d = \sqrt{1 - \frac{18^2}{19 \cdot 46}} \cdot \sqrt{46}$$

$$d = \sqrt{46 - \frac{324}{19}}$$

$$\boxed{d = \sqrt{\frac{550}{19}}}$$

MÉTODO 2:

$$\text{ÁREA} = |\vec{AB} \times \vec{AP}| = |\vec{AB}| . d$$

$$d = \frac{|\vec{AB} \times \vec{AP}|}{|\vec{AB}|}$$

$$\vec{AB} \times \vec{AP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 3 \\ 1 & 3 & 6 \end{vmatrix}$$

$$= \hat{i}(6-9) - \hat{j}(-18-3) + \hat{k}(-9-1)$$

$$= -3\hat{i} + 21\hat{j} - 10\hat{k}$$

$$|\vec{AB} \times \vec{AP}| = \sqrt{9 + 441 + 100}$$
$$= \sqrt{550}$$

$$\Rightarrow \boxed{d = \sqrt{\frac{550}{19}}}$$

3- $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{v}|}$ $\qquad \vec{u} = (-3, 0)$

$\vec{v} = (-2, y)$

$\vec{u}\cdot\vec{v} = (-3,0)\cdot(-2,y) = 6$

$|\vec{u}|\,|\vec{v}| = \sqrt{9}\cdot\sqrt{4+y^2} = 3\sqrt{y^2+4} \Rightarrow$

$\cos 60° = \dfrac{1}{2} = \dfrac{6}{3\sqrt{y^2+4}}$

$3\sqrt{y^2+4} = 12$

$\sqrt{y^2+4} = 4$

$y^2+4 = 16$

$y^2 = 12 \Rightarrow \boxed{y = \pm 2\sqrt{3}}$

4-

$\bar{A}REA = |\vec{a} \times \vec{b}|$

$$\bar{A}REA = \left\| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 5 & 1 & 3 \end{vmatrix} \right\|$$

$= |(1, -8, 1)|$

$= \sqrt{1+64+1} = \sqrt{66}$

$\boxed{\bar{A}REA = \sqrt{66}}$