Cálculo Diferencial e Integral - Prova 3

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Início: 19:00 - duração: 2:30 horas

Só serão consideradas as respostas que forem devidamente justificadas.
É proibido o uso de calculadoras, smartphones ou computadores.
É obrigatório simplificar todas as suas respostas.

Questão 01: Integrais

Resolva as seguintes integrais:

(a) (1,0) $\displaystyle\int_0^1 (x^3 - x)\,dx$

(b) (1,0) $\displaystyle\int_\pi^{2\pi} \cos(5x)\,dx$

(c) (1,0) $\displaystyle\int xe^{x^2}\,dx$

(d) (1,0) $\displaystyle\int \frac{2w^5 - w + 3}{w^2}\,dw$

(e) (1,0) $\displaystyle\int \frac{3t}{2 - 8t^2}\,dt$

(f) (1,0) $\displaystyle\int 2t^2(1 - 4t^3)\,dt$

(g) (1,0) $\displaystyle\int 4x\cos(2 - 3x)\,dx$

(h) (1,0) $\displaystyle\int \frac{dx}{3x - 1}$

(i) (1,0) $\displaystyle\int_0^1 e^x\sqrt{1 + e^x}\,dx$

(j) (1,0) $\displaystyle\int \operatorname{sen}(\pi t)\,dt$

GABARITO

1 - A) $\int_0^1 (x^3 - x)\,dx$

$$\int_0^1 (x^3 - x)\,dx = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_0^1$$
$$= \frac{1}{4} - \frac{1}{2}$$
$$= \boxed{-\frac{1}{4}}$$

B) $\int_\pi^{2\pi} \cos(5x)\,dx = \int \cos u \cdot \frac{du}{5}$

$u = 5x$
$du = 5\,dx$

$$= \frac{1}{5}\int \cos u\,du$$
$$= \frac{1}{5}\,\mathrm{sen}\,u$$
$$= \frac{1}{5}\,\mathrm{sen}(5x)\Big|_\pi^{2\pi}$$
$$= \frac{1}{5}\left[\mathrm{sen}(10\pi) - \mathrm{sen}(5\pi)\right]$$
$$= \boxed{0}$$

C) $\int x e^{x^2}\,dx = \int \frac{du}{2} \cdot e^u$

$u = x^2$
$du = 2x\,dx$

$$= \frac{1}{2}\int e^u\,du$$
$$= \frac{1}{2}e^u$$
$$= \boxed{\frac{1}{2}e^{x^2} + C}$$

D) $\int \frac{2w^5 - w + 3}{w^2}\,dw =$

$$= \int \left(2w^3 - \frac{1}{w} + 3w^{-2}\right)dw$$
$$= \frac{2w^4}{4} - \ln w + \frac{3w^{-1}}{-1}$$
$$= \boxed{\frac{w^4}{2} - \ln w - \frac{3}{w} + C}$$

E) $\int \frac{3t}{2 - 8t^2}\,dt = \int 3\cdot\left(-\frac{du}{16}\right)\cdot\frac{1}{u}$

$u = 2 - 8t^2$
$du = -16t\,dt$

$$= -\frac{3}{16}\int \frac{du}{u}$$
$$= -\frac{3}{16}\ln|u|$$
$$= \boxed{-\frac{3}{16}\ln|2 - 8t^2| + C}$$

F) $\int 2t^2(1 - 4t^3)\,dt$

$$= \int \left(2t^2 - 8t^5\right)dt$$
$$= \frac{2t^3}{3} - \frac{8t^6}{6}$$
$$= \boxed{\frac{2t^3}{3} - \frac{4t^6}{3} + C}$$

G) $\int 4x\cos(2-3x)\,dx = uv - \int v\,du$

$u = 4x$
$dv = \cos(2-3x)\,dx$
$du = 4\,dx$
$v = -\frac{1}{3}\text{sen}(2-3x)$

$= \frac{-4x}{3}\text{sen}(2-3x) + \int \frac{1}{3}\text{sen}(2-3x)\cdot 4\,dx$

$= \boxed{\frac{-4x\,\text{sen}(2-3x)}{3} + \frac{4}{9}\cos(2-3x) + C}$

H) $\int \frac{dx}{3x-1} = \int \frac{du}{3}\cdot\frac{1}{u}$

$u = 3x-1$
$du = 3\,dx$

$= \frac{1}{3}\int \frac{du}{u}$

$= \frac{1}{3}\ln u$

$= \boxed{\frac{1}{3}\ln(3x-1) + C}$

I) $\int_0^1 e^x\sqrt{1+e^x}\,dx = \int du\cdot\sqrt{u}$

$u = 1+e^x$
$du = e^x\,dx$

$= \int u^{1/2}\,du$

$= \frac{u^{3/2}}{3/2}$

$= \frac{2}{3}u^{3/2}$

$= \frac{2}{3}(1+e^x)^{3/2}\Big|_0^1$

$= \frac{2}{3}(1+e)^{3/2} - \frac{2}{3}(1+1)^{3/2}$

$= \boxed{\frac{2}{3}\left[(1+e)^{3/2} - 2\sqrt{2}\right]}$

J) $\int \text{sen}(\pi t)\,dt = \int \text{sen}\,u\,\frac{du}{\pi}$

$u = \pi t$
$du = \pi\,dt$

$= \frac{1}{\pi}\int \text{sen}\,u\,du$

$= \frac{-1}{\pi}\cos u$

$= \boxed{-\frac{\cos(\pi t)}{\pi} + C}$