Cálculo Diferencial e Integral
Lista de Problemas 3.3

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Questão 01

Calcule a integral fazendo a substituição dada.

(a) $\displaystyle\int \cos 3x\, dx, \ \ u = 3x$

(b) $\displaystyle\int x^2 \sqrt{x^3 + 1}\, dx, \ \ u = x^3 + 1$

(c) $\displaystyle\int \cos^3 \theta \sin \theta\, d\theta, \ \ u = \cos \theta$

Questão 02

Calcule a integral indefinida.

(a) $\displaystyle\int x \sin (x^2)\, dx$

(b) $\displaystyle\int (3x - 2)^{20}\, dx$

(c) $\displaystyle\int (x + 1)\sqrt{2x + x^2}\, dx$

(d) $\displaystyle\int \frac{dx}{5 - 3x}$

(e) $\displaystyle\int \sin \pi t\, dt$

(f) $\displaystyle\int \frac{e^u}{(1 - e^u)^2}\, du$

(g) $\displaystyle\int \frac{a + bx^2}{\sqrt{3ax + bx^3}}\, dx$

(h) $\displaystyle\int \frac{(\ln x)^2}{x}\, dx$

(i) $\displaystyle\int \sec^2\theta \tan^3\theta\, d\theta$

(j) $\displaystyle\int e^x\sqrt{1+e^x}\, dx$

(k) $\displaystyle\int (x^2+1)(x^3+3x)^4\, dx$

(l) $\displaystyle\int 5^t \sin 5^t\, dt$

(m) $\displaystyle\int e^{\tan x}\sec^2 x\, dx$

(n) $\displaystyle\int \frac{\cos x}{\sin^2 x}\, dx$

(o) $\displaystyle\int \sqrt{\cot x}\,\csc^2 x\, dx$

(p) $\displaystyle\int \sinh^2 x\cosh x\, dx$

(q) $\displaystyle\int \frac{\sin 2x}{1+\cos^2 x}\, dx$

(r) $\displaystyle\int \cot x\, dx$

(s) $\displaystyle\int \frac{dx}{\sqrt{1-x^2}\,\arcsin x}$

(t) $\displaystyle\int \frac{1+x}{1+x^2}\, dx$

(u) $\displaystyle\int x(2x+5)^8\, dx$

Questão 03

Avalie a integral definida.

(a) $\displaystyle\int_0^1 \cos(\pi t/2)\, dt$

(b) $\displaystyle\int_0^1 \sqrt[3]{1 + 7x}\, dx$

(c) $\displaystyle\int_0^\pi \sec^2(t/4)\, dt$

(d) $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\, dx$

(e) $\displaystyle\int_{-\pi/4}^{\pi/4} \left(x^3 + x^4 \tan x\right) dx$

(f) $\displaystyle\int_0^{13} \frac{dx}{\sqrt[3]{(1 + 2x)^2}}$

(g) $\displaystyle\int_0^a x\sqrt{x^2 + a^2}\, dx \quad a > 0$

(h) $\displaystyle\int_1^2 x\sqrt{x - 1}\, dx$

(i) $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

(j) $\displaystyle\int_0^1 \frac{e^z + 1}{e^z + z}\, dz$

(k) $\displaystyle\int_0^1 \frac{dx}{(1 + \sqrt{x})^4}$

Questão 04

Um tanque de armazenamento de petróleo sofre uma ruptura em $t = 0$ e o petróleo vaza do tanque a uma taxa de $r(t) = 100e^{-0,01t}$ litros por minuto. Quanto petróleo vazou na primeira hora?

Respostas

Questão 1

(a) $\frac{1}{3}\sin 3x + C$ (b) $\frac{2}{9}(x^3+1)^{3/2} + C$ (c) $-\frac{1}{4}\cos^4\theta + C$

Questão 2

(a) $-\frac{1}{2}\cos(x^2)+C$ (b) $\frac{1}{63}(3x-2)^{21}+C$ (c) $\frac{1}{3}(2x+x^2)^{3/2}+C$ (d) $-\frac{1}{3}\ln|5-3x|+C$ (e) $-(1/\pi)\cos\pi t+C$ (f) $\dfrac{1}{1-e^u}+C$ (g) $\frac{2}{3}\sqrt{3ax+bx^3}+C$ (h) $\frac{1}{3}(\ln x)^3+C$ (i) $\frac{1}{4}\tan^4\theta+C$ (j) $\frac{2}{3}(1+e^x)^{3/2}+C$ (k) $\frac{1}{15}(x^3+3x)^5+C$ (l) $-\dfrac{1}{\ln 5}\cos(5^t)+C$ (m) $e^{\tan x}+C$ (n) $-\dfrac{1}{\sin x}+C$ (o) $-\frac{2}{3}(\cot x)^{3/2}+C$ (p) $\frac{1}{3}\sinh^3 x+C$ (q) $-\ln(1+\cos^2 x)+C$ (r) $\ln|\sin x|+C$ (s) $-\ln|\arcsin x|+C$ (t) $\arctan x+\frac{1}{2}\ln(1+x^2)+C$ (u) $\frac{1}{40}(2x+5)^{10}-\frac{5}{36}(2x+5)^9+C$

Questão 3

(a) $2/\pi$ (b) $\frac{45}{28}$ (c) 4 (d) $e-\sqrt{e}$ (e) 0 (f) 3 (g) $\frac{1}{3}(2\sqrt{2}-1)a^3$ (h) $\frac{16}{15}$ (i) 2 (j) $\ln(e+1)$ (k) $\frac{1}{6}$

Questão 4

$\approx 4\,512$ L.